4. Equation of the Tangent to the Curve

Children of Mist – Ngāi Tūhoe

Te Urewera is a hill-country in the northern Hawke's Bay Region. Urewera is the historic home of Ngāi Tūhoe. Every *iwi*[1] in Aotearoa traces their origins to an ancestral *waka*[2], which came here from *Hawaiki*[3]. However, some tribes also have unique legends about their origins from the world around them. Some accounts say that Ngāi Tūhoe are descended from the mist of the Urewera Ranges: their ancestors were the Mist-Maiden who lured *Te Maunga*[4] to earth from the heavens. They had a son, who is the early founder of the iwi. To find out the name of the Mist-Maiden, the mother of the founder of the Ngāi Tūhoe, **find the equation of the tangent to the curve at the point indicated**. The equation and the associated letter will give the key to the answer. The first problem has been done for you.

Q 4.1 $f(x) = x^2 + 3x + 2$, at $(1,6)$ **E**

- Start with $y - y_1 = m\,(x - x_1)$
- Find the gradient function $f^{/}(x) = 2x + 3$
- Calculate $m = f^{/}(1) = 2 \times 1 + 3 = 5$
- Substitute $m = 5$ and $(1, 6)$: $y - 6 = 5(x - 1)$
- Rearrange the equation: $y = 5x + 1$
- So, $y = 5x + 1$ gives a letter E

Q 4.2 $f(x) = 4x^2 + 3x + 5$, at $(0,5)$ **P**

Q 4.3 $f(x) = x^2 + 2x + 1$, at $(2, 9)$ **O**

Q 4.4 $f(x) = x^2 - 6x + 15$, at $(3, 6)$ **A**

Q 4.5 $f(x) = 5x^2 - 3x - 20$, at $(1, -18)$ **K**

Q 4.6 $f(x) = 2x^3 - 3x^2 + 18$, at $(-2, -10)$ **U**

Q 4.7 $f(x) = 0.3x^{10} - 1.5x^2 + 2x + 7$, at $(0, 7)$ **R**

Q 4.8 $f(x) = 0.25x^4 - 2x^3 + 3x + 9$, at $(2, 3)$ **I**

Q 4.9 $f(x) = 2x^4 - 5x^2 + 3x - 9$, at $(1, -9)$ **N**	Q 4.10 $f(x) = -3x^2 - 5x + 9$, at $(0, 9)$ **H**
Q 4.11 $f(x) = 2x^2 + 3x + 7$, at $(1, 2)$ **I**	Q 4.12 $f(x) = x^2 - 3x + 8$, at $(2, 5)$ **H**
Q 4.13 $f(x) = x^3 + 2x - 12$, at $(1, 8)$ **U**	Q 4.14 $f(x) = x^4 - x^3$, at $(2, -5)$ **N**
Q 4.15 $f(x) = 3x^3 + 2x^2 + x - 1$, at $(0, 4)$ **G**	

$y = -5x + 9$	$y = -13x + 29$	$y = x - 10$	$y = 5x + 1$	$y = 3x + 5$	$y = 36x + 62$	$y = 7x - 25$	$y = 6x - 3$
			E				

$y = x + 3$	$y = 5x + 3$	$y = 2x + 7$	$y = 6$	$y = 20x - 45$	$y = x + 4$	$y = 7x - 5$

[1]*Iwi* – tribe; [2]*Waka* – canoe; [3]*Hawaiki* – the ancient homeland of Māori; [4]*Te Maunga* – mountain.

19. Rates of Change

The Meaning of Matapōuri

Ngāpuhi is New Zealand's largest iwi with more than 170,000 people according to the 2018 New Zealand census. The ancestral waka for Ngāpuhi is *Mataatua*, captained by Toroa and his younger brother Puhi. Many place names in Aotearoa are associated with the people from the *Mataatua*. One of them is *Matapōuri*, a coastal settlement in Northland, 35 km north-east of Whangarei. *Matapōuri* beach is 5th in the list of New Zealand's top 10 beaches. Some accounts say, Puhi named this beautiful place *Matapōuri*, which means gloomy, sad, dark. Why did Puhi do this? **To find out why, solve the problems below.** The letter beside each question and its answer **rounded to 2 significant figures** will give the key to the puzzle. Two questions have been done for you.

Q 19.1 Calculate the rate at which the area of a circle is changing with respect to the radius when the radius of a circle is 5cm. **R**

- Area of a circle $A = \pi r^2$
- *Rate of change* $\frac{dA}{dr} = [\pi r^2]' = 2\pi r$
- When $r = 5$, $2\pi r = 2\pi \times 5 = 31\ cm$

Q 19.2 Calculate the rate at which the surface area of a sphere is changing with respect to the radius when the radius of a sphere is 3cm. **T**

- Surface area of a sphere, $A = 4\pi r^2$
- ____
- ____
- ____
- ____

Q 19.3 Calculate the rate at which the volume of a sphere is changing with respect to the radius when the radius of a sphere is 2cm. **I**

Volume of a sphere $V = \frac{4\pi r^3}{3}$

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Q 19.4 Calculate the rate at which the volume of a cube is changing with respect to the length of a side when the side length of a cube is 4cm. **A**

Volume of a cube $V = x^3$

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Q 19.5 Calculate the rate at which the volume of a 5cm tall cylinder is changing with respect to the radius of a base when the radius of a cylinder is 3cm. **E**

Volume of a cylinder $V = \pi r^2 H$, $H = 5cm$

Q 19.6 Calculate the rate at which the volume of a 9cm tall cone is changing with respect to the radius of a base when the radius of a cone is 3cm. **H**

Volume of a cone $V = \frac{\pi r^2 H}{3}$, $H = 9\ cm$

Q 19.7 The radius, measured in metres, of a circular oil spillage after *t* minutes of the accident, can be modelled by formula $r = 5t + 7$. Find the rate of change of the circular area of the oil spillage after 4 minutes. **D**

- Area of a circle, $A = \pi r^2$, $r = 5t + 7$
- $A = \pi(5t + 7)^2$
- $\pi(25t^2 + 70t + 49)$
- $A = 25\pi t^2 + 70\pi t + 49\pi$
- *Rate of change with respect of time* $= \frac{dA}{dt} = A^{/}$
- $\frac{dA}{dt} = [25\pi t^2 + 70\pi t + 49\pi]^{/} = 50\pi t + 70\pi + 0$
- *When* $t = 4$, $50\pi \times 4 + 70 \times \pi = 848 m^2/min$

Q 19.8 As a weather balloon ascends into air, its radius slowly increases before it bursts out. After *t* hours, the radius of the balloon, in metres, can be modelled by the formula $r = 0.05t + 2$. Find the rate of change of the volume of the balloon after 8 hours. **N**

Volume of a sphere, $V = \frac{4\pi r^3}{3}$, $r = 0.05t + 2$

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57 cm^2	94 cm^2	5 cm		48 cm^2	3.6 m^3/hr	850 m^2/min	94 cm^2	850 m^2/min	5 cm	50 cm^2	3.6 m^3/hr
			L			D		D			

75 cm	57 cm^2	94 cm^2	31 cm	94 cm^2	2 cm	48 cm^2	75 cm	1 cm	3.6 m^3/hr	50 cm^2		57 cm^2	75 cm
			R								G		

ANSWERS:

Topic Number	
4	HINEPŪKOHURANGI
19	HE LANDED IN THERE AT NIGHT