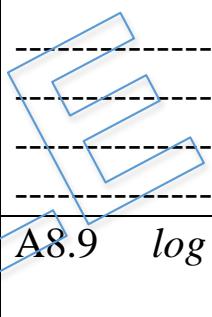
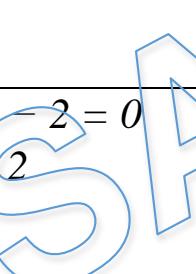
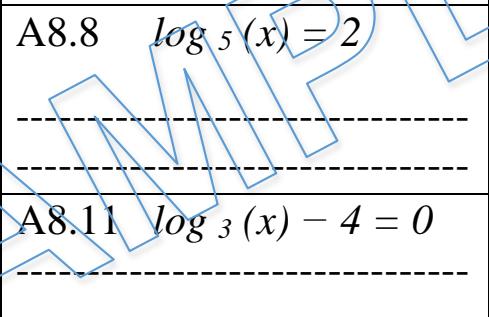
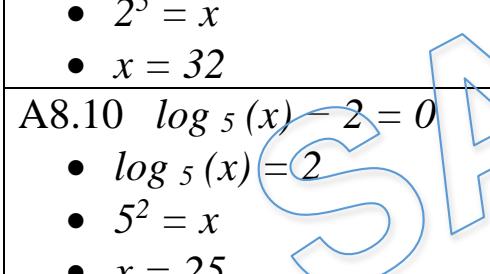


A8. Simple Logarithmic Equations

Solve the following equations

A8.1 $\log_x(49) = 2$ <ul style="list-style-type: none"> • $x^2 = 49$ • $x = 7 \text{ or } x = -7$ • $x \text{ cannot be negative}$ • $x = 7$ 	A8.2 $\log_x(64) = 2$ <hr/> <hr/> <hr/> <hr/>	A8.3 $\log_x(16) = 2$ <hr/> <hr/> <hr/> <hr/>
A8.4 $\log_x(8) = 3$ <hr/> <hr/> <hr/> <hr/>	A8.5 $\log_x(64) = 3$ <hr/> <hr/> <hr/> <hr/>	A8.6 $\log_x(32) = 5$  <hr/> <hr/> <hr/> <hr/>
A8.7 $\log_2(x) = 5$ <ul style="list-style-type: none"> • $2^5 = x$ • $x = 32$ 	A8.8 $\log_5(x) = 2$ 	A8.9 $\log_7(x) = 0$ <hr/> <hr/> <hr/> <hr/>
A8.10 $\log_5(x) - 2 = 0$ <ul style="list-style-type: none"> • $\log_5(x) = 2$ • $5^2 = x$ • $x = 25$ 	A8.11 $\log_3(x) - 4 = 0$ <hr/> <hr/> <hr/> <hr/>	A8.12 $\log_2(x) + 1 = 0$ <hr/> <hr/> <hr/> <hr/>
A8.13 $\log_3(2x - 1) = 2$ <ul style="list-style-type: none"> • $2x - 1 = 3^2$ • $2x - 1 = 9$ • $2x = 10$ • $x = 5$ 	A8.14 $\log_5(3x - 1) = 3$ <hr/> <hr/> <hr/> <hr/>	A8.15 $\log_7(5m + 4) = 2$ <hr/> <hr/> <hr/> <hr/>
A8.16 $\log_5(2n - 1) = 1$ <hr/> <hr/> <hr/> <hr/>	A8.17 $\log_2(5n + 2) = -1$ <hr/> <hr/> <hr/> <hr/>	A8.18 $\log_2(5y + 1) = -3$ <hr/> <hr/> <hr/> <hr/>

A8. Simple Logarithmic Equations. Solutions

Solve the following equations

A8.1 $\log_x(49) = 2$ <ul style="list-style-type: none"> • $x^2 = 49$ • $x = 7 \text{ or } x = -7$ • $x \text{ cannot be negative}$ • $x = 7$ 	A8.2 $\log_x(64) = 2$ <ul style="list-style-type: none"> • $x^2 = 64$ • $x = 8 \text{ or } x = -8$ • $x \text{ cannot be negative}$ • $x = 8$ 	A8.3 $\log_x(16) = 2$ <ul style="list-style-type: none"> • $x^2 = 16$ • $x = 4 \text{ or } x = -4$ • $x \text{ cannot be negative}$ • $x = 4$
A8.4 $\log_x(8) = 3$ <ul style="list-style-type: none"> • $x^3 = 8$ • $x = \sqrt[3]{8} = 2$ 	A8.5 $\log_x(64) = 3$ <ul style="list-style-type: none"> • $x^3 = 64$ • $x = \sqrt[3]{64} = 4$ 	A8.6 $\log_x(32) = 5$ <ul style="list-style-type: none"> • $x^5 = 32$ • $x = \sqrt[5]{32} = 2$
A8.7 $\log_2(x) = 5$ <ul style="list-style-type: none"> • $2^5 = x$ • $x = 32$ 	A8.8 $\log_5(x) = 2$ <ul style="list-style-type: none"> • $5^2 = x$ • $x = 25$ 	A8.9 $\log_7(x) = 0$ <ul style="list-style-type: none"> • $7^0 = x$ • $x = 1$
A8.10 $\log_5(x) - 2 = 0$ <ul style="list-style-type: none"> • $\log_5(x) = 2$ • $5^2 = x$ • $x = 25$ 	A8.11 $\log_3(x) - 4 = 0$ <ul style="list-style-type: none"> • $\log_3(x) = 4$ • $3^4 = x$ • $x = 81$ 	A8.12 $\log_2(x) + 1 = 0$ <ul style="list-style-type: none"> • $\log_2(x) = -1$ • $2^{-1} = x$ • $x = 0.5$
A8.13 $\log_3(2x-1) = 2$ <ul style="list-style-type: none"> • $2x-1 = 3^2$ • $2x-1 = 9$ • $2x = 10$ • $x = 5$ 	A8.14 $\log_5(3x-1) = 3$ <ul style="list-style-type: none"> • $3x-1 = 5^3$ • $3x-1 = 125$ • $3x = 126$ • $x = 42$ 	A8.15 $\log_7(5m+4) = 2$ <ul style="list-style-type: none"> • $5m+4 = 7^2$ • $5m+4 = 49$ • $5m = 45$ • $m = 9$
A8.16 $\log_5(2n-1) = 1$ <ul style="list-style-type: none"> • $2n-1 = 5^1$ • $2n-1 = 5$ • $2n = 6$ • $n = 3$ 	A8.17 $\log_2(5n+2) = -1$ <ul style="list-style-type: none"> • $5n+2 = 2^{-1}$ • $5n+2 = 0.5$ • $5n = -1.5$ • $n = -0.3$ 	A8.18 $\log_2(5y+1) = -3$ <ul style="list-style-type: none"> • $5y+1 = 2^{-3}$ • $5y+1 = 0.125$ • $5y = -0.875$ • $y = -0.175$

M7. Solving Logarithmic Equations

Solve the problems below.

M7.1 Find the value of $\log_2 15$

- Let $\log_2 15 = x$
- $2^x = 15$
- $\log[2^x] = \log 15$
- $x \log 2 = \log 15$
- $x = \frac{\log 15}{\log 2}$

M7.2 Find the value of $\log_3 25$

M7.3 Find the value of $\log_5 35$

M7.4 Find the value of $\log_{\frac{1}{2}} 8$

M7.5 Solve: $\log_x[x^2 + 2x - 8] = 2$

- $x^2 + 2x - 8 = x^2$
- $x^2 + 2x - 8 - x^2 = 0$
- $2x - 8 = 0$
- $2x = 8, x = 4$

M7.6 Solve: $\log_x[x^2 + x - 7] = 2$

M/E7.7 $\log_x[2x^2 + x - 6] = 2$

M/E7.8 $\log_x[3x^2 - 5x - 12] = 2$

M7.9 $\log_x[2x^2 - 5x + 6] = 2$

M/E7.10 $\log_x[x^2 - 4x + 4] = 1$

M7. Solving Logarithmic Equations. Solutions

Solve the problems below.

M7.1 Find the value of $\log_2 15$

- Let $\log_2 15 = x$
- $2^x = 15$
- $\log[2^x] = \log 15$
- $x \log 2 = \log 15$
- $x = \frac{\log 15}{\log 2} = 3.9069(4dp)$

M7.2 Find the value of $\log_3 25$

- Let $\log_3 25 = x$
- $3^x = 25$
- $\log[3^x] = \log 25$
- $x \log 3 = \log 25$
- $x = \frac{\log 25}{\log 3} = 2.9299(4dp)$

M7.3 Find the value of $\log_5 35$

- Let $\log_5 35 = x$
- $5^x = 35$
- $\log[5^x] = \log 35$
- $x \log 5 = \log 35$
- $x = \frac{\log 35}{\log 5} = 2.2091(4dp)$

M7.4 Find the value of $\log_{\frac{1}{2}}[8]$

- Let $\log_{\frac{1}{2}}[8] = x$
- $\left[\frac{1}{2}\right]^x = 8$
- $\log\left[\left(\frac{1}{2}\right)^x\right] = \log 8$
- $x \log\left(\frac{1}{2}\right) = \log 8$
- $x = \frac{\log 8}{\log\left(\frac{1}{2}\right)} = -3$

M7.5 Solve: $\log_x[x^2 + 2x - 8] = 2$

- $x^2 + 2x - 8 = x^2$
- $x^2 + 2x - 8 - x^2 = 0$
- $2x - 8 = 0$
- $2x = 8, x = 4$

M7.6 Solve: $\log_x[x^2 + x - 7] = 2$

- $x^2 + x - 7 = x^2$
- $x^2 + x - 7 - x^2 = 0$
- $x - 7 = 0$
- $x = 7$

M/E7.7 $\log_x[2x^2 + x - 6] = 2$

- $2x^2 + x - 6 = x^2$
- $2x^2 + x - 6 - x^2 = 0$
- $x^2 + x - 6 = 0$
- $x = 2 \text{ or } x = -3$
- Cannot be negative, so $x = 2$

M/E7.8 $\log_x[3x^2 - 5x - 12] = 2$

- $3x^2 - 5x - 12 = x^2$
- $3x^2 - 5x - 12 - x^2 = 0$
- $2x^2 - 5x - 12 = 0$
- $x = 4 \text{ or } x = -1.5$
- Cannot be negative, so $x = 4$

M7.9 $\log_x[2x^2 - 5x + 6] = 2$

- $2x^2 - 5x + 6 = x^2$
- $2x^2 - 5x + 6 - x^2 = 0$
- $x^2 - 5x + 6 = 0$
- $x = 2 \text{ or } x = 3$
- Both answers are valid

M/E7.10 $\log_x[x^2 - 4x + 4] = 1$

- $x^2 - 4x + 4 = x$
- $x^2 - 4x + 4 - x = 0$
- $x^2 - 5x + 4 = 0$
- $x = 1 \text{ or } x = 4$
- Base cannot be 1, so $x = 4$

E9. Using Quadratic Formula. Worksheet 3

Answer the questions below.

E9.1 Find the expression in terms of q for the sum of squares of the roots of the equation $3x^2 - 5x + q = 0$

- $ax^2 + bx + c = 0 \rightarrow$
- $a = 3, b = -5, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5^2 - 4 \times 3 \times q}}{2 \times 3}$
- $x = \frac{5 \pm \sqrt{25 - 12q}}{6}$
- $x_1 = \frac{5 + \sqrt{25 - 12q}}{6}, \rightarrow x_1^2 = \left[\frac{5 + \sqrt{25 - 12q}}{6} \right]^2$
- $x_2 = \frac{5 - \sqrt{25 - 12q}}{6}, \rightarrow x_2^2 = \left[\frac{5 - \sqrt{25 - 12q}}{6} \right]^2$
- $x_1^2 = \frac{5^2 + 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_1^2 = \frac{50 + 10\sqrt{25 - 12q} - 12q}{36}$
- $x_2^2 = \frac{5^2 - 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_2^2 = \frac{50 - 10\sqrt{25 - 12q} - 12q}{36}$
- $x_1^2 + x_2^2 = \frac{100 - 24q}{36} = \frac{25 - 6q}{9}$

E9.2 Find the expression in terms of q for the sum of squares of the roots of the equation $5x^2 - 4x + q = 0$

E9.3 Find the expression in terms of k for the sum of squares of the roots of the equation $5x^2 - kx + 1 = 0$

E9.4 Find the expression in terms of q for the sum of cubes of the roots of the equation $x^2 - 4x + q = 0$
Hint: $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$

E9. Using Quadratic Formula. Worksheet 3. Solutions

Answer the questions below.

E9.1 Find the expression in terms of q for the sum of squares of the roots of the equation $3x^2 - 5x + q = 0$

- $ax^2 + bx + c = 0 \rightarrow a = 3, b = -5, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5^2 - 4 \times 3 \times q}}{2 \times 3}$
- $x = \frac{5 \pm \sqrt{25 - 12q}}{6}$
- $x_1 = \frac{5 + \sqrt{25 - 12q}}{6}, \rightarrow x_1^2 = \left[\frac{5 + \sqrt{25 - 12q}}{6} \right]^2$
- $x_2 = \frac{5 - \sqrt{25 - 12q}}{6}, \rightarrow x_2^2 = \left[\frac{5 - \sqrt{25 - 12q}}{6} \right]^2$
- $x_1^2 = \frac{5^2 + 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_1^2 = \frac{50 + 10\sqrt{25 - 12q} - 12q}{36}$
- $x_2^2 = \frac{5^2 - 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_2^2 = \frac{50 - 10\sqrt{25 - 12q} - 12q}{36}$
- $x_1^2 + x_2^2 = \frac{100 - 24q}{36} = \frac{25 - 6q}{9}$

E9.2 Find the expression in terms of q for the sum of squares of the roots of the equation $5x^2 - 4x + q = 0$

- $x^2 + bx + c = 0 \rightarrow a = 5, b = -4, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{4^2 - 4 \times 5 \times q}}{2 \times 5}$
- $x = \frac{4 \pm \sqrt{16 - 20q}}{10}$
- $x_1 = \frac{4 + \sqrt{16 - 20q}}{10}, \rightarrow x_1^2 = \left[\frac{4 + \sqrt{16 - 20q}}{10} \right]^2$
- $x_2 = \frac{4 - \sqrt{16 - 20q}}{10}, \rightarrow x_2^2 = \left[\frac{4 - \sqrt{16 - 20q}}{10} \right]^2$
- $x_1^2 = \frac{4^2 + 2 \times 4 \times \sqrt{16 - 20q} + 16 - 20q}{100} \rightarrow$
- $x_1^2 = \frac{32 + 8\sqrt{16 - 20q} - 20q}{100}$
- $x_2^2 = \frac{4^2 - 2 \times 4 \times \sqrt{16 - 20q} + 16 - 20q}{100} \rightarrow$
- $x_2^2 = \frac{32 - 8\sqrt{16 - 20q} - 20q}{100}$
- $x_1^2 + x_2^2 = \frac{64 - 40q}{100} = \frac{16 - 10q}{25}$

E9.3 Find the expression in terms of k for the sum of squares of the roots of the equation $5x^2 - kx + 1 = 0$

- $ax^2 + bx + c = 0 \rightarrow a = 5, b = -k, c = 1$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{k \pm \sqrt{k^2 - 4 \times 5 \times 1}}{2 \times 5}$
- $x = \frac{k \pm \sqrt{k^2 - 20}}{10}$
- $x_1 = \frac{k + \sqrt{k^2 - 20}}{10}, \rightarrow x_1^2 = \left[\frac{k + \sqrt{k^2 - 20}}{10} \right]^2$
- $x_2 = \frac{k - \sqrt{k^2 - 20}}{10}, \rightarrow x_2^2 = \left[\frac{k - \sqrt{k^2 - 20}}{10} \right]^2$
- $x_1^2 = \frac{k^2 + 2 \times k \times \sqrt{k^2 - 20} + k^2 - 20}{100} \rightarrow$
- $x_1^2 = \frac{2k^2 + 2k\sqrt{k^2 - 20} - 20}{100}$
- $x_2^2 = \frac{2k^2 - 2k\sqrt{k^2 - 20} - 20}{100}$
- $x_1^2 + x_2^2 = \frac{4k^2 - 40}{100} = \frac{k^2 - 10}{25}$

E9.4 Find the expression in terms of q for the sum of cubes of the roots of the equation $x^2 - 4x + q = 0$

- Hint: $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$
- Let m and n are the roots
 - $(x - m)(x - n) = 0$
 - $x^2 - mx - nx + mn = 0$
 - $x^2 - (m + n)x + mn = 0$
 - $x^2 - 4x + q = 0, \rightarrow$
 - $m + n = 4$ and $mn = q$
 - $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$
 - $m^3 + n^3 = (m + n)[(m + n)^2 - 3mn]$
 - $m^3 + n^3 = 4(4^2 - 3q) = 64 - 12q$

