A8. Simple Logarithmic Equations

Solve the following equations

A8.1 $\log_x (49) = 2$ • $x^2 = 49$ • $x = 7$ or $x = -7$ • x cannot be negative • $x = 7$	A8.2 $\log_x (64) = 2$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.3 $\log_x (16) = 2$ -------------------------------- -------------------------------- -------------------------------- --------------------------------
A8.4 $\log_x (8) = 3$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.5 $\log_x (64) = 3$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.6 $\log_x (32) = 5$ -------------------------------- -------------------------------- -------------------------------- --------------------------------
A8.7 $\log_2 (x) = 5$ • $2^5 = x$ • $x = 32$	A8.8 $\log_5 (x) = 2$ -------------------------------- --------------------------------	A8.9 $\log_7 (x) = 0$ -------------------------------- --------------------------------
A8.10 $\log_5 (x) - 2 = 0$ • $\log_5 (x) = 2$ • $5^2 = x$ • $x = 25$	A8.11 $\log_3 (x) - 4 = 0$ -------------------------------- -------------------------------- --------------------------------	A8.12 $\log_2 (x) + 1 = 0$ -------------------------------- -------------------------------- --------------------------------
A8.13 $\log_3 (2x - 1) = 2$ • $2x - 1 = 3^2$ • $2x - 1 = 9$ • $2x = 10$ • $x = 5$	A8.14 $\log_5 (3x - 1) = 3$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.15 $\log_7 (5m + 4) = 2$ -------------------------------- -------------------------------- -------------------------------- --------------------------------
A8.16 $\log_5 (2n - 1) = 1$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.17 $\log_2 (5n + 2) = -1$ -------------------------------- -------------------------------- -------------------------------- --------------------------------	A8.18 $\log_2 (5y + 1) = -3$ -------------------------------- -------------------------------- -------------------------------- --------------------------------

A8. Simple Logarithmic Equations. Solutions

Solve the following equations

A8.1 $\log_x(49) = 2$ • $x^2 = 49$ • $x = 7$ or $x = -7$ • x cannot be negative • $x = 7$	A8.2 $\log_x(64) = 2$ • $x^2 = 64$ • $x = 8$ or $x = -8$ • x cannot be negative • $x = 8$	A8.3 $\log_x(16) = 2$ • $x^2 = 16$ • $x = 4$ or $x = -4$ • x cannot be negative • $x = 4$
A8.4 $\log_x(8) = 3$ • $x^3 = 8$ • $x = \sqrt[3]{8} = 2$	A8.5 $\log_x(64) = 3$ • $x^3 = 64$ • $x = \sqrt[3]{64} = 4$	A8.6 $\log_x(32) = 5$ • $x^5 = 32$ • $x = \sqrt[5]{32} = 2$
A8.7 $\log_2(x) = 5$ • $2^5 = x$ • $x = 32$	A8.8 $\log_5(x) = 2$ • $5^2 = x$ • $x = 25$	A8.9 $\log_7(x) = 0$ • $7^0 = x$ • $x = 1$
A8.10 $\log_5(x) - 2 = 0$ • $\log_5(x) = 2$ • $5^2 = x$ • $x = 25$	A8.11 $\log_3(x) - 4 = 0$ • $\log_3(x) = 4$ • $3^4 = x$ • $x = 81$	A8.12 $\log_2(x) + 1 = 0$ • $\log_2(x) = -1$ • $2^{-1} = x$ • $x = 0.5$
A8.13 $\log_3(2x - 1) = 2$ • $2x - 1 = 3^2$ • $2x - 1 = 9$ • $2x = 10$ • $x = 5$	A8.14 $\log_5(3x - 1) = 3$ • $3x - 1 = 5^3$ • $3x - 1 = 125$ • $3x = 126$ • $x = 42$	A8.15 $\log_7(5m + 4) = 2$ • $5m + 4 = 7^2$ • $5m + 4 = 49$ • $5m = 45$ • $m = 9$
A8.16 $\log_5(2n - 1) = 1$ • $2n - 1 = 5^1$ • $2n - 1 = 5$ • $2n = 6$ • $n = 3$	A8.17 $\log_2(5n + 2) = -1$ • $5n + 2 = 2^{-1}$ • $5n + 2 = 0.5$ • $5n = -1.5$ • $n = -0.3$	A8.18 $\log_2(5y + 1) = -3$ • $5y + 1 = 2^{-3}$ • $5y + 1 = 0.125$ • $5y = -0.875$ • $y = -0.175$

M7. Solving Logarithmic Equations

Solve the problems below.

M7.1 Find the value of $log_2 15$ • Let $log_2 15 = x$ • 2^x=15 • $log[2^x] = log15$ • $xlog2 = log15$ • $x = \frac{log15}{log2}$	M7.2 Find the value of $log_3 25$
M7.3 Find the value of $log_5 35$	M7.4 Find the value of $log_{\frac{1}{2}}[8]$
M7.5 Solve: $log_x[x^2 + 2x - 8] = 2$ • $x^2 + 2x - 8 = x^2$ • $x^2 + 2x - 8 - x^2 = 0$ • $2x - 8 = 0$ • $2x = 8\,, x = 4$	M7.6 Solve: $log_x[x^2 + x - 7] = 2$
M/E7.7 $log_x[2x^2 + x - 6] = 2$	M/E7.8 $log_x[3x^2 - 5x - 12] = 2$
M7.9 $log_x[2x^2 - 5x + 6] = 2$	M/E7.10 $log_x[x^2 - 4x + 4] = 1$

M7. Solving Logarithmic Equations. Solutions

Solve the problems below.

M7.1 Find the value of $log_2 15$ • Let $log_2 15 = x$ • 2^x=15 • $log[2^x] = log15$ • $xlog2 = log15$ • $x = \frac{log15}{log2} = 3.9069(4dp)$	M7.2 Find the value of $log_3 25$ • Let $log_3 25 = x$ • 3^x=25 • $log[3^x] = log25$ • $xlog3 = log25$ • $x = \frac{log25}{log3} = 2.9299\ (4dp)$
M7.3 Find the value of $log_5 35$ • Let $log_5 35 = x$ • 5^x=35 • $log[5^x] = log35$ • $xlog5 = log35$ • $x = \frac{log35}{log5} = 2.2091(4dp)$	M7.4 Find the value of $log_{\frac{1}{2}}[8]$ • Let $log_{\frac{1}{2}}[8] = x$ • $\left[\frac{1}{2}\right]^x$=8 • $log\left[\left(\frac{1}{2}\right)^x\right] = log8$ • $xlog\left(\frac{1}{2}\right) = log8$ • $x = \frac{log8}{log\left(\frac{1}{2}\right)} = -3$
M7.5 Solve: $log_x[x^2 + 2x - 8] = 2$ • $x^2 + 2x - 8 = x^2$ • $x^2 + 2x - 8 - x^2 = 0$ • $2x - 8 = 0$ • $2x = 8\ , x = 4$	M7.6 Solve: $log_x[x^2 + x - 7] = 2$ • $x^2 + x - 7 = x^2$ • $x^2 + x - 7 - x^2 = 0$ • $x - 7 = 0$ • $x = 7$
M/E7.7 $log_x[2x^2 + x - 6] = 2$ • $2x^2 + x - 6 = x^2$ • $2x^2 + x - 6 - x^2 = 0$ • $x^2 + x - 6 = 0$ • $x = 2\ or\ x = -3$ • Cannot be negative, so $x = 2$	M/E7.8 $log_x[3x^2 - 5x - 12] = 2$ • $3x^2 - 5x - 12 = x^2$ • $3x^2 - 5x - 12 - x^2 = 0$ • $2x^2 - 5x - 12 = 0$ • $x = 4\ or\ x = -1.5$ • Cannot be negative, so $x = 4$
M7.9 $log_x[2x^2 - 5x + 6] = 2$ • $2x^2 - 5x + 6 = x^2$ • $2x^2 - 5x + 6 - x^2 = 0$ • $x^2 - 5x + 6 = 0$ • $x = 2\ or\ x = 3$ • Both answers are valid	M/E7.10 $log_x[x^2 - 4x + 4] = 1$ • $x^2 - 4x + 4 = x$ • $x^2 - 4x + 4 - x = 0$ • $x^2 - 5x + 4 = 0$ • $x = 1\ or\ x = 4$ • Base cannot be 1, so $x = 4$

E9. Using Quadratic Formula. Worksheet 3

Answer the questions below.

E9.1 *Find the expression in terms of* q *for the sum of squares of the roots of the equation* $3x^2 - 5x + q = 0$

- $ax^2 + bx + c = 0 \rightarrow$
- $a = 3, b = -5, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5^2 - 4 \times 3 \times q}}{2 \times 3}$
- $x = \frac{5 \pm \sqrt{25 - 12q}}{6}$
- $x_1 = \frac{5 + \sqrt{25 - 12q}}{6}, \rightarrow x_1^2 = \left[\frac{5 + \sqrt{25 - 12q}}{6}\right]^2$
- $x_2 = \frac{5 - \sqrt{25 - 12q}}{6}, \rightarrow x_2^2 = \left[\frac{5 - \sqrt{25 - 12q}}{6}\right]^2$
- $x_1^2 = \frac{5^2 + 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_1^2 = \frac{50 + 10\sqrt{25 - 12q} - 12q}{36}$
- $x_2^2 = \frac{5^2 - 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_2^2 = \frac{50 - 10\sqrt{25 - 12q} - 12q}{36}$
- $x_1^2 + x_2^2 = \frac{100 - 24q}{36} = \frac{25 - 6q}{9}$

E9.2 *Find the expression in terms of* q *for the sum of squares of the roots of the equation* $5x^2 - 4x + q = 0$

E9.3 *Find the expression in terms of* k *for the sum of squares of the roots of the equation* $5x^2 - kx + 1 = 0$

E9.4 *Find the expression in terms of* q *for the sum of cubes of the roots of the equation* $x^2 - 4x + q = 0$

Hint: $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$

E9. Using Quadratic Formula. Worksheet 3. Solutions

Answer the questions below.

E9.1 *Find the expression in terms of q for the sum of squares of the roots of the equation $3x^2 - 5x + q = 0$*

- $ax^2 + bx + c = 0 \rightarrow \quad a = 3, b = -5, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5^2 - 4 \times 3 \times q}}{2 \times 3}$
- $x = \frac{5 \pm \sqrt{25 - 12q}}{6}$
- $x_1 = \frac{5 + \sqrt{25 - 12q}}{6}, \rightarrow x_1^2 = \left[\frac{5 + \sqrt{25 - 12q}}{6}\right]^2$
- $x_2 = \frac{5 - \sqrt{25 - 12q}}{6}, \rightarrow x_2^2 = \left[\frac{5 - \sqrt{25 - 12q}}{6}\right]^2$
- $x_1^2 = \frac{5^2 + 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_1^2 = \frac{50 + 10\sqrt{25 - 12q} - 12q}{36}$
- $x_2^2 = \frac{5^2 - 2 \times 5 \times \sqrt{25 - 12q} + 25 - 12q}{36} \rightarrow$
- $x_2^2 = \frac{50 - 10\sqrt{25 - 12q} - 12q}{36}$
- $x_1^2 + x_2^2 = \frac{100 - 24q}{36} = \frac{25 - 6q}{9}$

E9.2 *Find the expression in terms of q for the sum of squares of the roots of the equation $5x^2 - 4x + q = 0$*

- $x^2 + bx + c = 0 \rightarrow \quad a = , b = -4, c = q$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{4^2 - 4 \times 5 \times q}}{2 \times 5}$
- $x = \frac{4 \pm \sqrt{16 - 20q}}{10}$
- $x_1 = \frac{4 + \sqrt{16 - 20q}}{10}, \rightarrow x_1^2 = \left[\frac{4 + \sqrt{16 - 20q}}{10}\right]^2$
- $x_2 = \frac{4 - \sqrt{16 - 20q}}{10}, \rightarrow x_2^2 = \left[\frac{4 - \sqrt{16 - 20q}}{10}\right]^2$
- $x_1^2 = \frac{4^2 + 2 \times 4 \times \sqrt{16 - 20q} + 16 - 20q}{100} \rightarrow$
- $x_1^2 = \frac{32 + 8\sqrt{16 - 20q} - 20q}{100}$
- $x_2^2 = \frac{4^2 - 2 \times 4 \times \sqrt{16 - 20q} + 16 - 20q}{100} \rightarrow$
- $x_2^2 = \frac{32 - 8\sqrt{16 - 20q} - 20q}{100}$
- $x_1^2 + x_2^2 = \frac{64 - 40q}{100} = \frac{16 - 10q}{25}$

E9.3 *Find the expression in terms of k for the sum of squares of the roots of the equation $5x^2 - kx + 1 = 0$*

- $ax^2 + bx + c = 0 \rightarrow \; a = 5, b = -k, c = 1$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{k \pm \sqrt{k^2 - 4 \times 5 \times 1}}{2 \times 5}$
- $x = \frac{k \pm \sqrt{k^2 - 20}}{10}$
- $x_1 = \frac{k + \sqrt{k^2 - 20}}{10}, \rightarrow x_1^2 = \left[\frac{k + \sqrt{k^2 - 20}}{10}\right]^2$
- $x_2 = \frac{k - \sqrt{k^2 - 20}}{10}, \rightarrow x_2^2 = \left[\frac{k - \sqrt{k^2 - 20}}{10}\right]^2$
- $x_1^2 = \frac{k^2 + 2 \times k \times \sqrt{k^2 - 20} + k^2 - 20}{100} \rightarrow$
- $x_1^2 = \frac{2k^2 + 2k\sqrt{k^2 - 20} - 20}{100}$
- $x_2^2 = \frac{2k^2 - 2k\sqrt{k^2 - 20} - 20}{100}$
- $x_1^2 + x_2^2 = \frac{4k^2 - 40}{100} = \frac{k^2 - 10}{25}$

E9.4 *Find the expression in terms of q for the sum of cubes of the roots of the equation $x^2 - 4x + q = 0$*
Hint: $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$

- *Let m and n are the roots*
- $(x - m)(x - n) = 0$
- $x^2 - mx - nx + mn = 0$
- $x^2 - (m + n)x + mn = 0$
- $x^2 - 4x + q = 0, \rightarrow$
- $m + n = 4 \;\; and \;\; mn = q$
- $m^3 + n^3 = (m + n)(m^2 - mn + n^2)$
- $m^3 + n^3 = (m + n)[(m + n)^2 - 3mn]$
- $m^3 + n^3 = 4(4^2 - 3q) = 64 - 12q$